1.10 ⋆ The particle's polar angle is $\phi = \omega t$, so $x = R\cos(\omega t)$ and $y = R\sin(\omega t)$ or

$$\mathbf{r} = \hat{\mathbf{x}}\, R\cos(\omega t) + \hat{\mathbf{y}}\, R\sin(\omega t).$$

Differentiating, we find that $\dot{\mathbf{r}} = -\hat{\mathbf{x}}\,\omega R\sin(\omega t) + \hat{\mathbf{y}}\,\omega R\cos(\omega t)$ and then

$$\ddot{\mathbf{r}} = -\hat{\mathbf{x}}\,\omega^2 R\cos(\omega t) - \hat{\mathbf{y}}\,\omega^2 R\sin(\omega t) = -\omega^2\mathbf{r} = -\omega^2 R\,\hat{\mathbf{r}}.$$

That is, the acceleration is antiparallel to the radius vector and has magnitude $a = \omega^2 R = v^2/R$, the well known centripetal acceleration.

1.17 ⋆⋆ **(a)** Let us start with the x component of $\mathbf{r}\times(\mathbf{u}+\mathbf{v})$. From the definition (1.9), we see that

$$[\mathbf{r}\times(\mathbf{u}+\mathbf{v})]_{\mathbf{x}} = r_y(u_z+v_z) - r_z(u_y+v_y) = (r_y u_z - r_z u_y) + (r_y v_z - r_z v_y) = (\mathbf{r}\times\mathbf{u})_x + (\mathbf{r}\times\mathbf{v})_x\,.$$

Since the y and z components follow in the same way, we conclude that $\mathbf{r}\times(\mathbf{u}+\mathbf{v}) = \mathbf{r}\times\mathbf{u}+\mathbf{r}\times\mathbf{v}$.

(b) Starting again from (1.9), we find for the x component

$$\frac{d}{dt}(\mathbf{r}\times\mathbf{s})_x = \frac{d}{dt}(r_y s_z - r_z s_y) = \left(r_y\frac{ds_z}{dt} - r_z\frac{ds_y}{dt}\right) + \left(\frac{dr_y}{dt}s_z - \frac{dr_z}{dt}s_y\right) = \left(\mathbf{r}\times\frac{d\mathbf{s}}{dt} + \frac{d\mathbf{r}}{dt}\times\mathbf{s}\right)_x\,.$$

This is the x component of the desired identity. Since the y and z components follow in exactly the same way, our proof is complete.

1.19 ⋆⋆ $\dfrac{d}{dt}[\mathbf{a}\cdot(\mathbf{v}\times\mathbf{r})] = \dfrac{d\mathbf{a}}{dt}\cdot(\mathbf{v}\times\mathbf{r}) + \mathbf{a}\cdot\dfrac{d}{dt}(\mathbf{v}\times\mathbf{r}) = \dot{\mathbf{a}}\cdot(\mathbf{v}\times\mathbf{r}) + \mathbf{a}\cdot(\dot{\mathbf{v}}\times\mathbf{r}+\mathbf{v}\times\dot{\mathbf{r}})$.
The final term $\mathbf{a}\cdot(\mathbf{v}\times\dot{\mathbf{r}})$ is zero because $\dot{\mathbf{r}} = \mathbf{v}$ and $\mathbf{v}\times\mathbf{v} = 0$. The second to last term is $\mathbf{a}\cdot(\mathbf{a}\times\mathbf{r}) = \mathbf{0}$, because $\mathbf{a}\times\mathbf{r}$ is perpendicular to $\mathbf{a}$, so their scalar product is zero. This leaves us with the requested identity.

1.29 $\star$ When we write out Equations (1.25) and (1.26) for the four particles, we get four equations:

$$\begin{aligned}
\dot{\mathbf{p}}_1 &= (\text{net force on particle 1}) = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_{14} + \mathbf{F}_1^{\text{ext}} \\
\dot{\mathbf{p}}_2 &= (\text{net force on particle 2}) = \mathbf{F}_{21} + \mathbf{F}_{23} + \mathbf{F}_{24} + \mathbf{F}_2^{\text{ext}} \\
\dot{\mathbf{p}}_3 &= (\text{net force on particle 3}) = \mathbf{F}_{31} + \mathbf{F}_{32} + \mathbf{F}_{34} + \mathbf{F}_3^{\text{ext}} \\
\dot{\mathbf{p}}_4 &= (\text{net force on particle 4}) = \mathbf{F}_{41} + \mathbf{F}_{42} + \mathbf{F}_{43} + \mathbf{F}_4^{\text{ext}} .
\end{aligned}$$

Adding these four equations, we find for $\dot{\mathbf{P}} = \dot{\mathbf{p}}_1 + \dot{\mathbf{p}}_2 + \dot{\mathbf{p}}_3 + \dot{\mathbf{p}}_4$,

$$\begin{aligned}
\dot{\mathbf{P}} = &(\mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_{14}) + (\mathbf{F}_{21} + \mathbf{F}_{23} + \mathbf{F}_{24}) + (\mathbf{F}_{31} + \mathbf{F}_{32} + \mathbf{F}_{34}) + (\mathbf{F}_{41} + \mathbf{F}_{42} + \mathbf{F}_{43}) \\
&+ (\mathbf{F}_1^{\text{ext}} + \mathbf{F}_2^{\text{ext}} + \mathbf{F}_3^{\text{ext}} + \mathbf{F}_4^{\text{ext}}).
\end{aligned} \tag{ii}$$

This corresponds to Equation (1.27). The twelve terms on the first line of the right side can be rearranged to give

$$(\mathbf{F}_{12} + \mathbf{F}_{21}) + (\mathbf{F}_{13} + \mathbf{F}_{31}) + (\mathbf{F}_{14} + \mathbf{F}_{41}) + (\mathbf{F}_{23} + \mathbf{F}_{32}) + (\mathbf{F}_{24} + \mathbf{F}_{42}) + (\mathbf{F}_{34} + \mathbf{F}_{43}) = 0$$

since each of the six pairs is zero by Newton's third law. Thus Equation (ii) for $\dot{\mathbf{P}}$ reduces to

$$\dot{\mathbf{P}} = \mathbf{F}_1^{\text{ext}} + \mathbf{F}_2^{\text{ext}} + \mathbf{F}_3^{\text{ext}} + \mathbf{F}_4^{\text{ext}} = \mathbf{F}^{\text{ext}}$$

which is the required Equation (1.29).

1.30 $\star$ Since mass 2 is at rest, the initial total momentum is just $\mathbf{P}_{\text{in}} = m_1\mathbf{v}$. The final total momentum is $\mathbf{P}_{\text{fin}} = (m_1 + m_2)\mathbf{v}'$. Equating these two and solving for $\mathbf{v}'$, we find that $\mathbf{v}' = \mathbf{v}m_1/(m_1 + m_2)$.

1.35 $\star$ In the absence of air resistance, the net force on the ball is $\mathbf{F} = m\mathbf{g}$, and with the given choice of axes, $\mathbf{g} = (0, 0, -g)$. Thus Newton's second law, $\mathbf{F} = m\ddot{\mathbf{r}}$, implies that $\ddot{\mathbf{r}} = \mathbf{g}$, or

$$\ddot{x} = 0, \qquad \ddot{y} = 0, \qquad \text{and} \qquad \ddot{z} = -g.$$

The initial velocity has components $v_{\text{o}x} = v_{\text{o}} \cos\theta$, $v_{\text{o}y} = 0$, and $v_{\text{o}z} = v_{\text{o}} \sin\theta$, and we can choose the initial position to be the origin. The first of the above equations can be integrated once to give $\dot{x} = v_{\text{o}x}$, and again to give $x(t) = v_{\text{o}x}t$. In the same way, the y equation gives $y(t) = 0$, and the z equation gives $z(t) = v_{\text{o}z}t - \frac{1}{2}gt^2$. The ball returns to the ground when $z(t) = 0$ which gives $t = 2v_{\text{o}z}/g$. Substituting this time into the expression for $x(t)$ gives the range, range $= 2v_{\text{o}x}v_{\text{o}z}/g$.

1.38 ⋆ The two forces on the puck are its weight $m\mathbf{g}$ and the normal force $\mathbf{N}$ of the incline. With the suggested choice of axes, $\mathbf{N} = (0, 0, N)$ and $\mathbf{g} = (0, -g\sin\theta, -g\cos\theta)$. Thus Newton's second law reads

$$m\ddot{\mathbf{r}} = \mathbf{N} + m\mathbf{g} \quad \text{or} \quad \begin{cases} m\ddot{x} = 0 \\ m\ddot{y} = -mg\sin\theta \\ m\ddot{z} = N - mg\cos\theta \end{cases}$$

By integrating the y equation twice, we find that $y = v_{oy}t - \frac{1}{2}gt^2\sin\theta$. Thus the time to return to the line $y = 0$ is $t = 2v_{oy}/(g\sin\theta)$ and the distance from O at that time is $x = v_{ox}t = 2v_{ox}v_{oy}/(g\sin\theta)$.

1.39 ⋆⋆ $x = v_o t\cos\theta - \frac{1}{2}gt^2\sin\phi$, $y = v_o t\sin\theta - \frac{1}{2}gt^2\cos\phi$, $z = 0$. When the ball returns to the plane, y is 0, which implies that $t = 2v_o\sin\theta/(g\cos\phi)$. Substituting this time into x and using a couple of trig identities yields the claimed answer for the range R. To find the maximum range, differentiate R with respect to θ and set the derivative equal to zero. This gives $\theta = (\pi - 2\phi)/4$, and substitution into R (plus another trig identity) yields the claimed value of R_{max}.

1.46 ⋆⋆ **(a)** As seen in the inertial frame $\mathcal{S}$ the puck moves in a straight line with $\phi = 0$ and $r = R - v_o t$

(b) As seen in $\mathcal{S}'$, $r' = r = R - v_o t$ and $\phi' = \phi - \omega t = -\omega t$. This path is sketched in the answer to Problem 1.27. Initially, the puck moves inward with speed v_o but also downward with speed ωR. It curves to its right, passing through the center and continuing to curve to the right until it slides off the turntable.

1.49 ⋆⋆ There are two forces on the puck, the net normal force of the two cylinders and the force of gravity. So $\mathbf{F} = N\hat{\boldsymbol{\rho}} - mg\hat{\mathbf{z}}$. Since the puck is confined between the cylinders, $\rho = R$, a constant. The three components of $\mathbf{F} = m\mathbf{a}$ are:

$$\begin{array}{lll} F_\rho = m(\ddot{\rho} - \rho\dot{\phi}^2) & \text{or} & N = -mR\dot{\phi}^2 \\ F_\phi = m(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}) & \text{or} & 0 = mR\ddot{\phi} \\ F_z = m\ddot{z} & \text{or} & -mg = m\ddot{z}. \end{array}$$

The ρ equation tells us the magnitude and direction (inward) of the normal force. The ϕ equation tells us that $\dot{\phi}$ is constant. (This is actually conservation of angular momentum.) Thus $\dot{\phi} = \omega$, a constant, and hence $\phi = \phi_o + \omega t$. The puck moves around the cylinder at a constant rate ω. The z equation tells us that $\dot{z} = v_{oz} - gt$ and hence that $z = z_o + v_{oz}t - \frac{1}{2}gt^2$. That is, the vertical motion is precisely that of a body in free fall. The resulting path is a helix of downward increasing pitch.
